Probability of Having the Same Birthday

Here is the typically how the question is asked: "If 30 people are at a party, what is the probability of at least two people having the exact same birthday?"

Your first response is--"That scenario seems totally unlikely. There are 365 days in a non leap year so it must be very improbable that 2 people will have the same birthday."

There are several ideas from probability occurring here.

Let's consider the case where each person has a different birthday. (The compliment of this situation is the one in which at least two people have the same birthday. We will worry about that in a minute.)

The first person, they can have 365 different birthdays. The second person can only have 364 different birthdays since we want everyone to have a different birthday. The third person has only a possibility of 363 days for a birthday. This can happen then in $365 \cdot 364 \cdot 363$ different ways.

How many different ways total with no restrictions?: $365 \cdot 365 \cdot 365$ Each person could be born on any day of the year.

The probability then is $\left(\frac{365 \cdot 364 \cdot 363}{365^3}\right)$ which equals 0.9917958341152186. This is the probability that three people will have different birthdays. Therefore, the probability that at least 2 will have the SAME birthday is 1- 0.9917958341152186 = 0.008204 . This is the compliment.

Therefore there is a 0.82% chance that from three people at least 2 will have the same birthday.

Now to answer the question of 30 people.

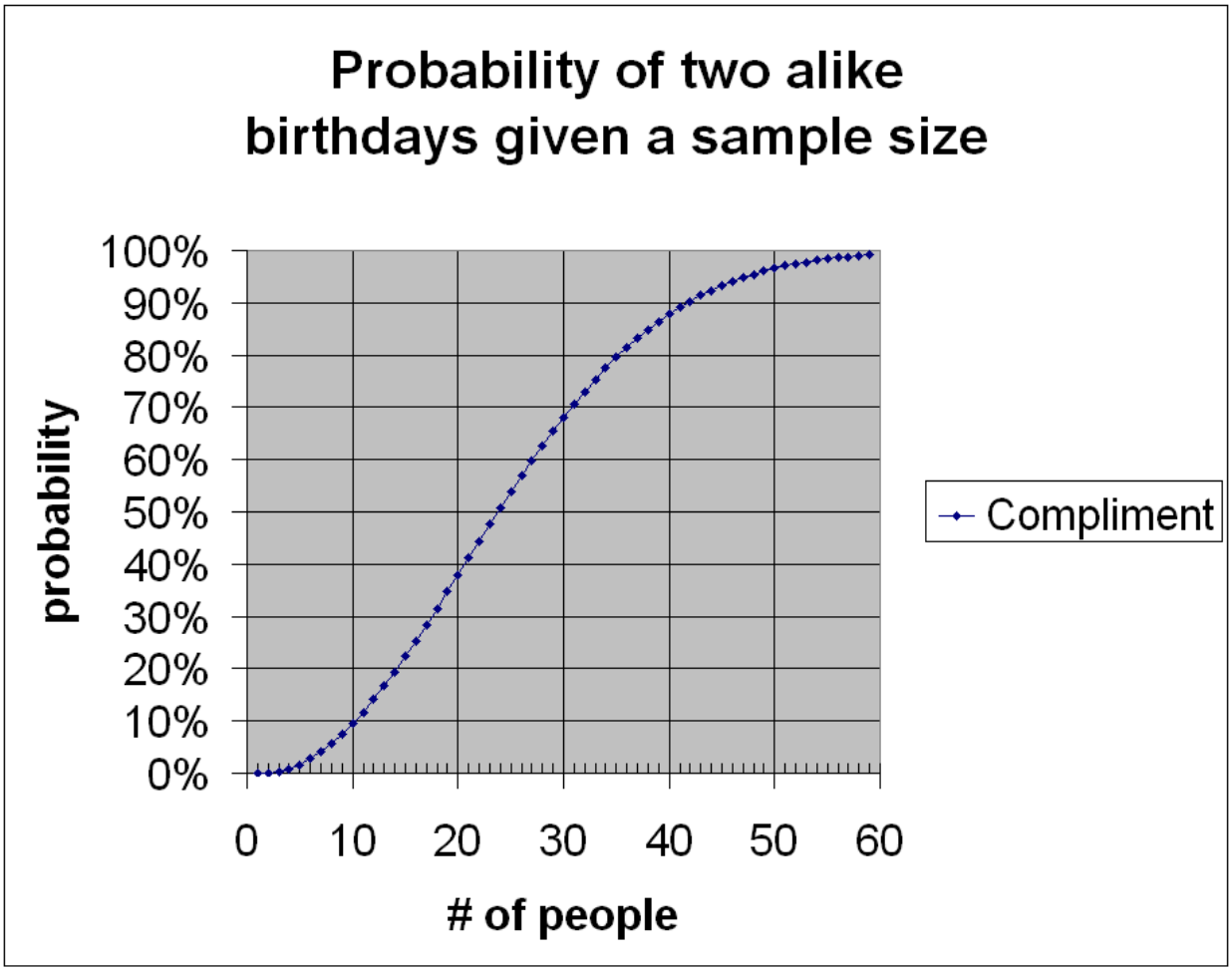
Follow the same pattern. This will be the probability that at least 2 of the 30 people have the same birthday.

Therefore, $1-\left(\frac{365 \cdot 364 \cdot 363 \cdot \ldots \cdot 337 \cdot 336}{365^{30}}\right)$ This value is 70.6 %

Here is a complete table of values: Click here for the graph

Sample Size	Prob of different birthdays	Compliment Prob of at least 2 having the same birhtday
1	1.0000000000	0.0000%
2	0.9972602740	0.2740%
3	0.9917958341	0.8204%
4	0.9836440875	1.6356%
5	0.9728644263	2.7136%
6	0.9595375164	4.0462%
7	0.9437642969	5.6236%
8	0.9256647076	7.4335%
9	0.9053761661	9.4624%
10	0.8830518223	11.6948%
11	0.8588586217	14.1141%
12	0.8329752112	16.7025%
13	0.8055897248	19.4410%
14	0.7768974880	22.3103%
15	0.7470986802	25.2901%
16	0.7163959947	28.3604%
17	0.6849923347	31.5008%
18	0.6530885821	34.6911%
19	0.6208814740	37.9119%
20	0.5885616164	41.1438%
21	0.5563116648	44.3688%
22	0.5243046923	47.5695%
23	0.4927027657	50.7297%
24	0.4616557421	53.8344%
25	0.4313002960	56.8700%
26	0.4017591799	59.8241%
27	0.3731407177	62.6859%
28	0.3455385277	65.4461%
29	0.3190314625	68.0969%
30	0.2936837573	70.6316%
31	0.2695453663	73.0455%

32	0.2466524721	75.3348%
33	0.2250281458	77.4972%
34	0.2046831354	79.5317%
35	0.1856167611	81.4383%
36	0.1678178936	83.2182%
37	0.1512659918	84.8734%
38	0.1359321789	86.4068%
39	0.1217803356	87.8220%
40	0.1087681902	89.1232%
41	0.0968483885	90.3152%
42	0.0859695284	91.4030%
43	0.0760771443	92.3923%
44	0.0671146314	93.2885%
45	0.0590241005	94.0976%
46	0.0517471566	94.8253%
47	0.0452255972	95.4774%
48	0.0394020271	96.0598%
49	0.0342203907	96.5780%
50	0.0296264204	97.0374%
51	0.0255680067	97.4432%
52	0.0219954907	97.8005%
53	0.0188618865	98.1138%
54	0.0161230372	98.3877%
55	0.0137377112	98.6262%
56	0.0116676451	98.8332%
57	0.0098775407	99.0122%
58	0.0083350206	99.1665%


Probability of two alike birthdays given a sample size
probability
100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
0
10
20
30
40
50
60
# of people
Compliment